Introduction

绪　论

- Measurement
  测量
- Significant Figures
  有效数字
- Error and Uncertainty
  误差及不确定度
- Data processing
  数据处理

Measurement

测　　量

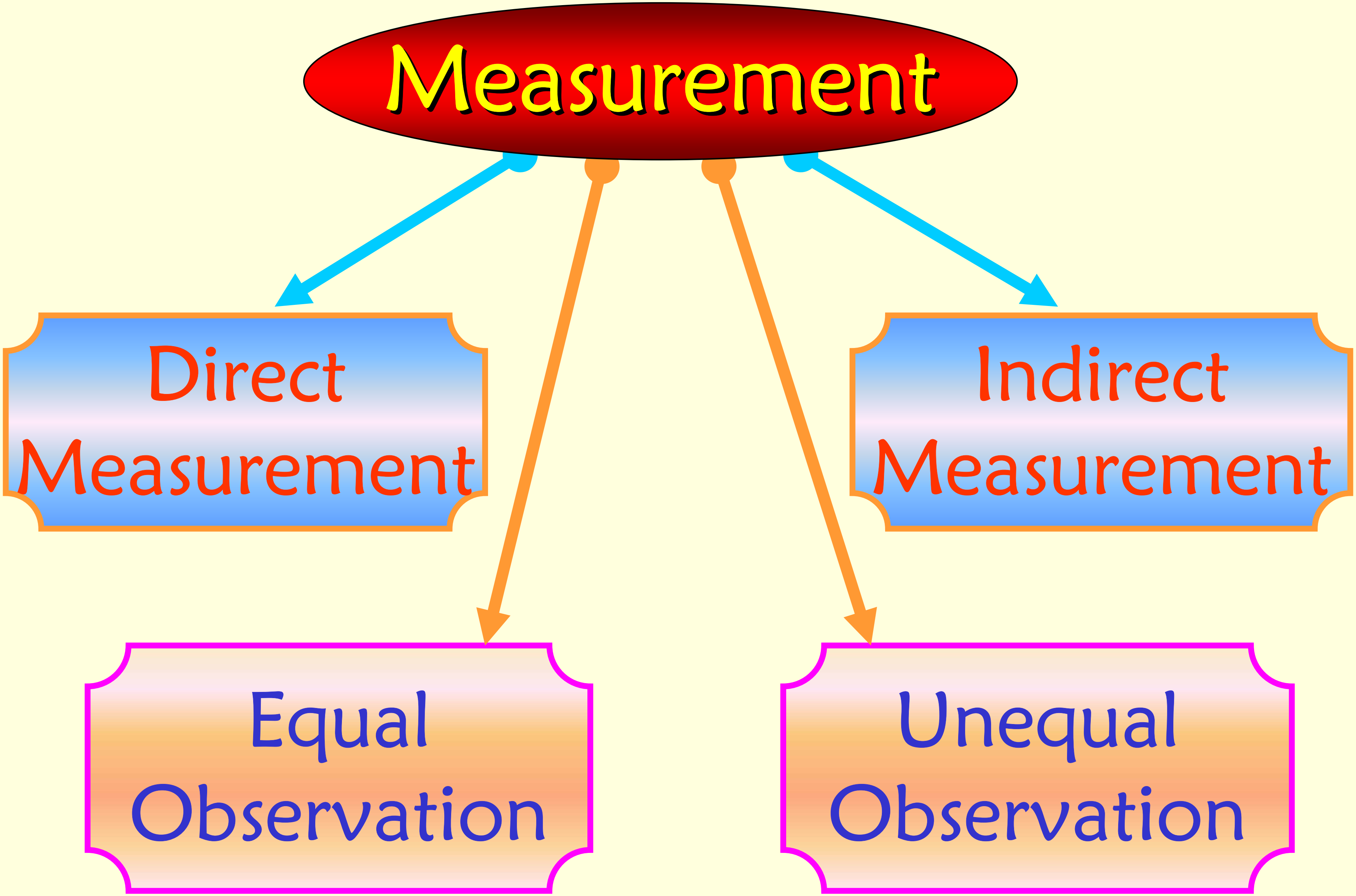
Physical quantities物理量

All the quantities describing states or movements of the matter are called physical quantities.

The value of physical quantities can be determined through measurement.

What is measurement?

- Measurement of the physical quantity is a fundamental operation of physical experiments. In fact measuring is a direct or indirect quantitative comparison between the physical quantity of the object to be measured and the one selected to be standard measuring unit through specified method with some devices.


Measurement
Direct Measurement
Indirect Measurement
Equal Observation
Unequal Observation

Direct measurement 直接测量

- Direct measurement is the direct comparison made between quantity to be measured and the reference quantity (measuring devices or meters) in order to obtain metrical data and the corresponding physical quantities called directly measured quantities.
- Direct measurement is the base of other measurements.

Indirect measurement 间接测量

- Some quantities cannot be directly measured by devices or meters therefore they need to be calculated on the base of some other quantities being able to be directly measured and having specified functional relation with them. Similarly the corresponding quantities are called indirectly measured quantities.

Equal and Unequal Observations

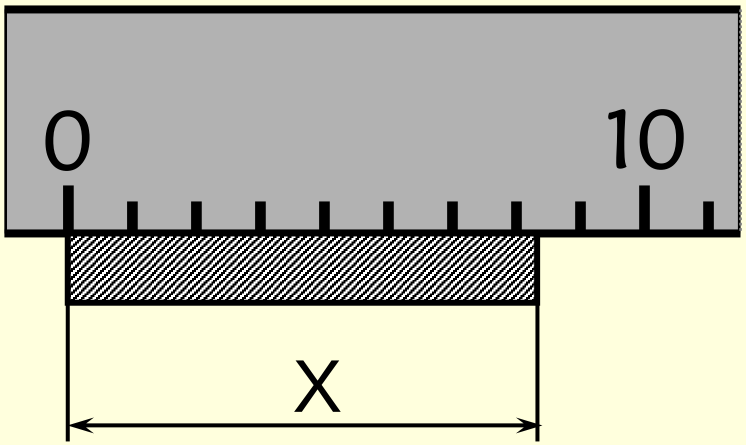
等精度和非等精度测量

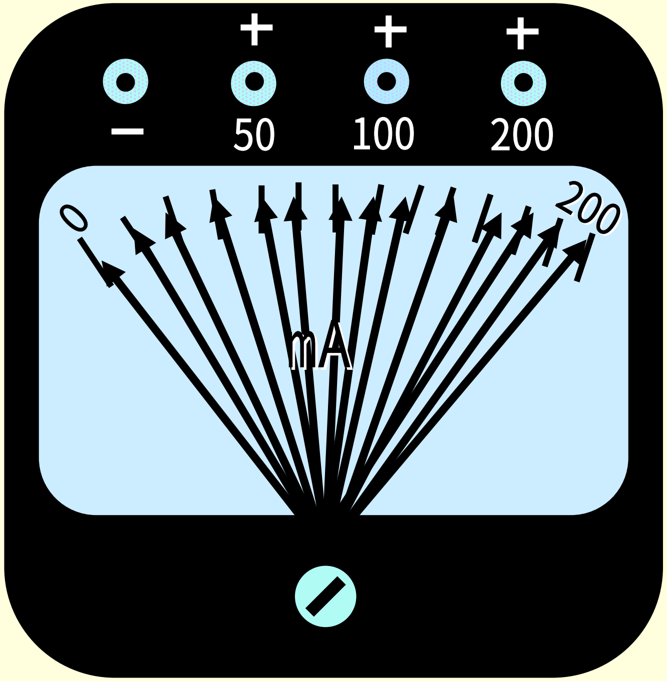
If a physical quantity is repeatedly measured for many times under the same conditions such as by the same operator using the same instruments in the same circumstance and then a series of observations is obtained as x_1, x_2,…, x_n. The different measured values in a series are reliable to the same extent and such kind of measurements is called equal observations. If even one condition mentioned above is changed the kind of measurements is called unequal observations.

Methods for Measurements 测量方法

- Relative method 比较法
- Compensatory approach 补偿法
- Measurement by magnification 放大法
- Simulation method 模拟法
- Conversion method 转换法
  - Electric means for non-electric quantity
  - Optical means for non-optical quantity
- Interferometry 干涉计量

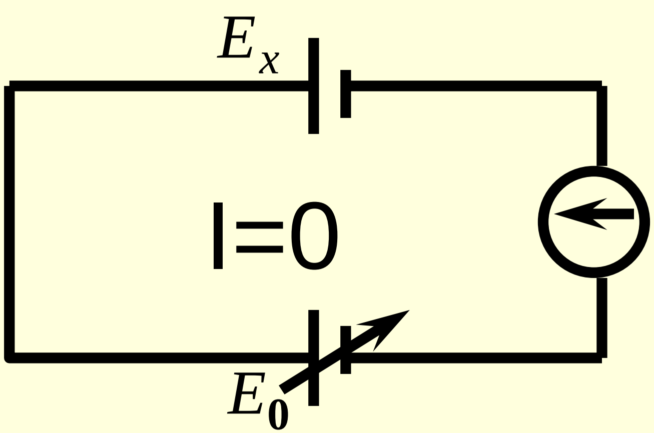
Relative method



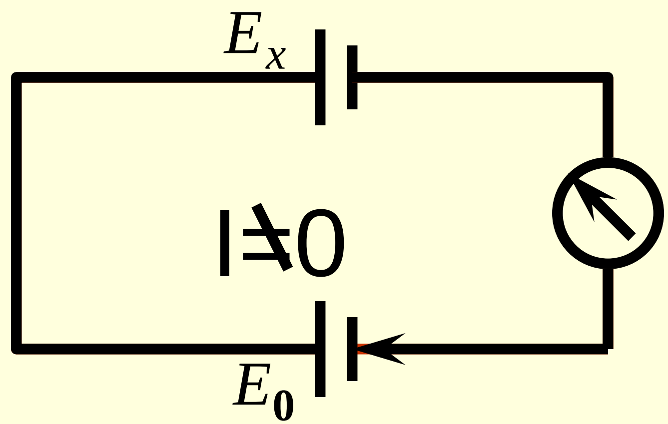


Compensatory approach

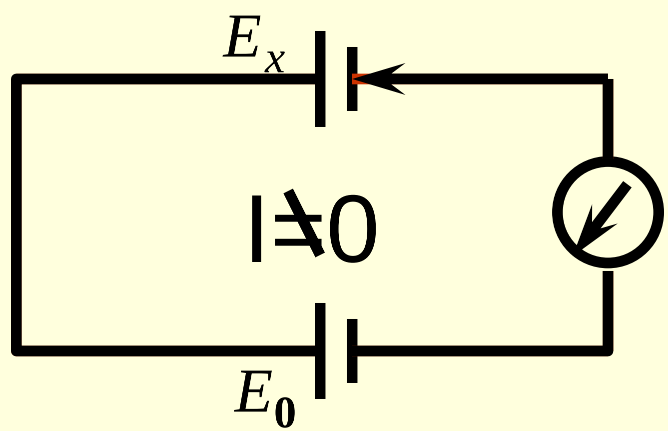
$E_x = E_0$



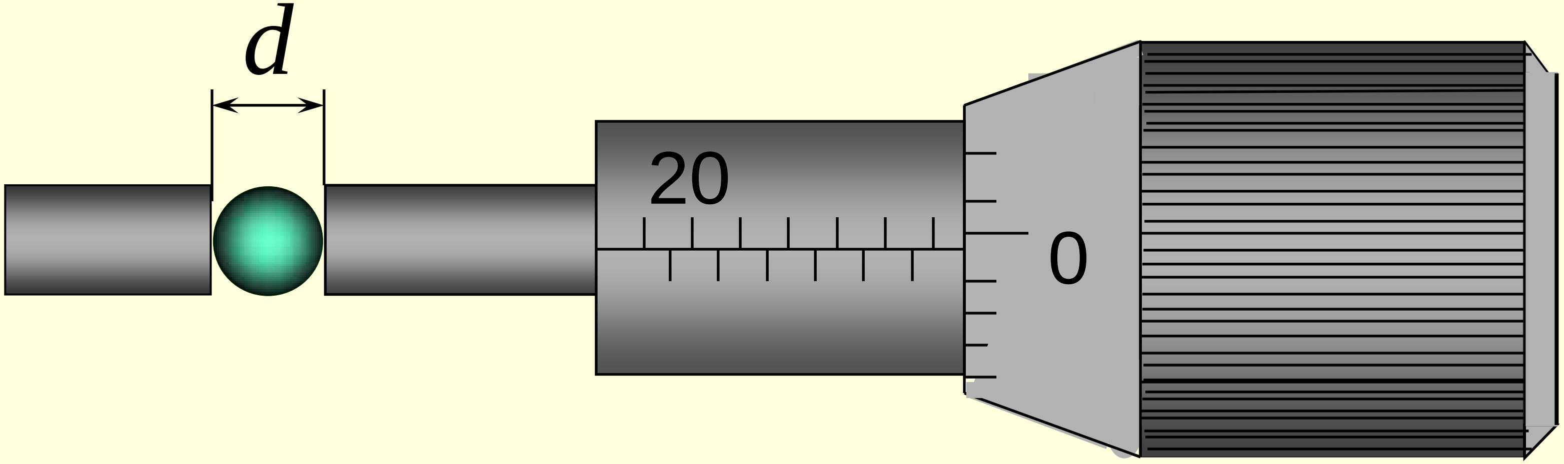
$E_x < E_0$



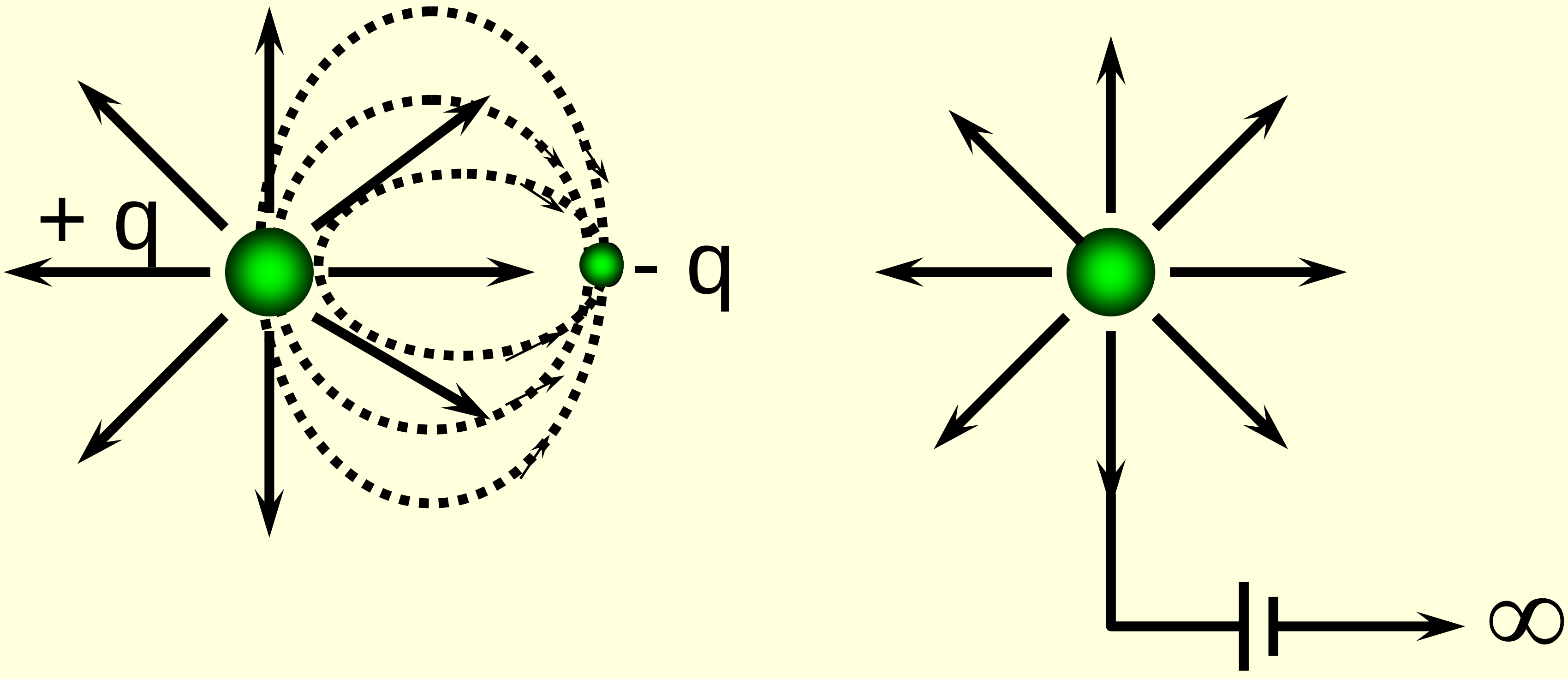
$E_x > E_0$



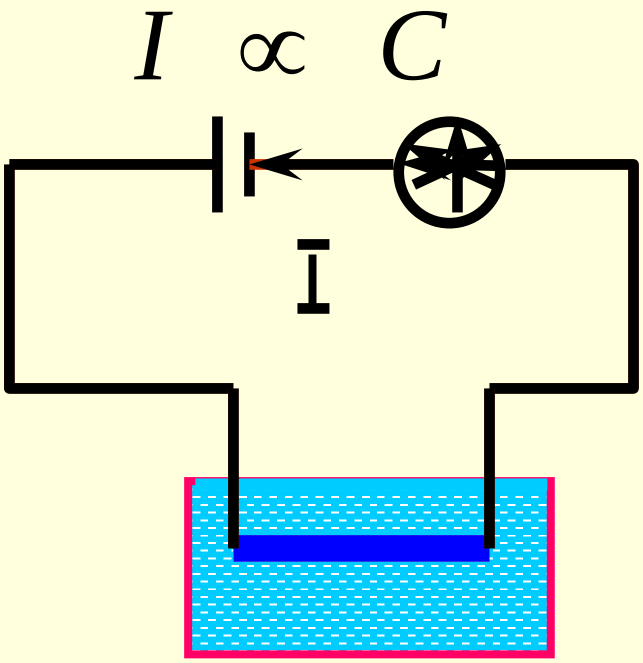
Measurement by magnification

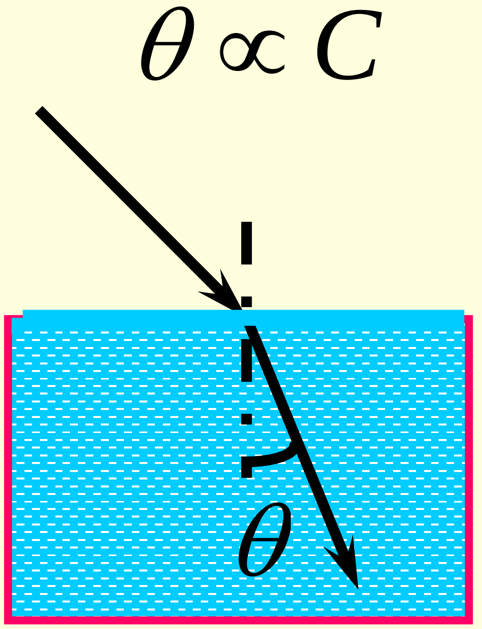


Simulation method

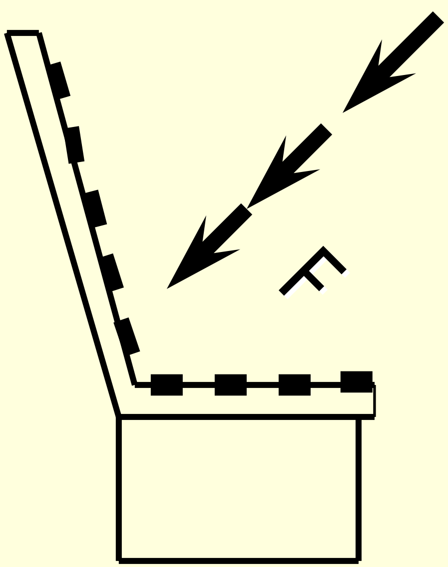


Conversion method

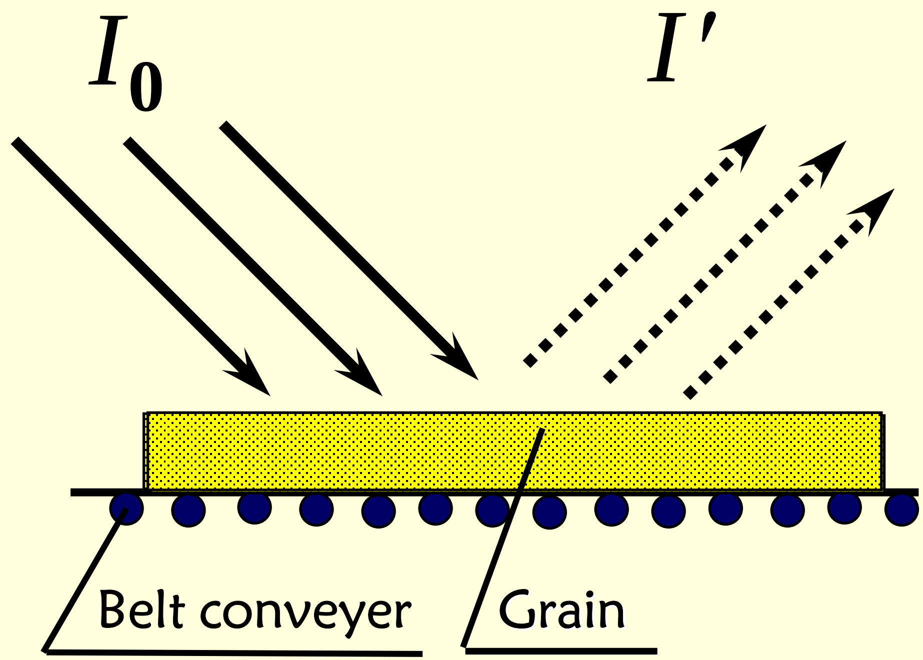


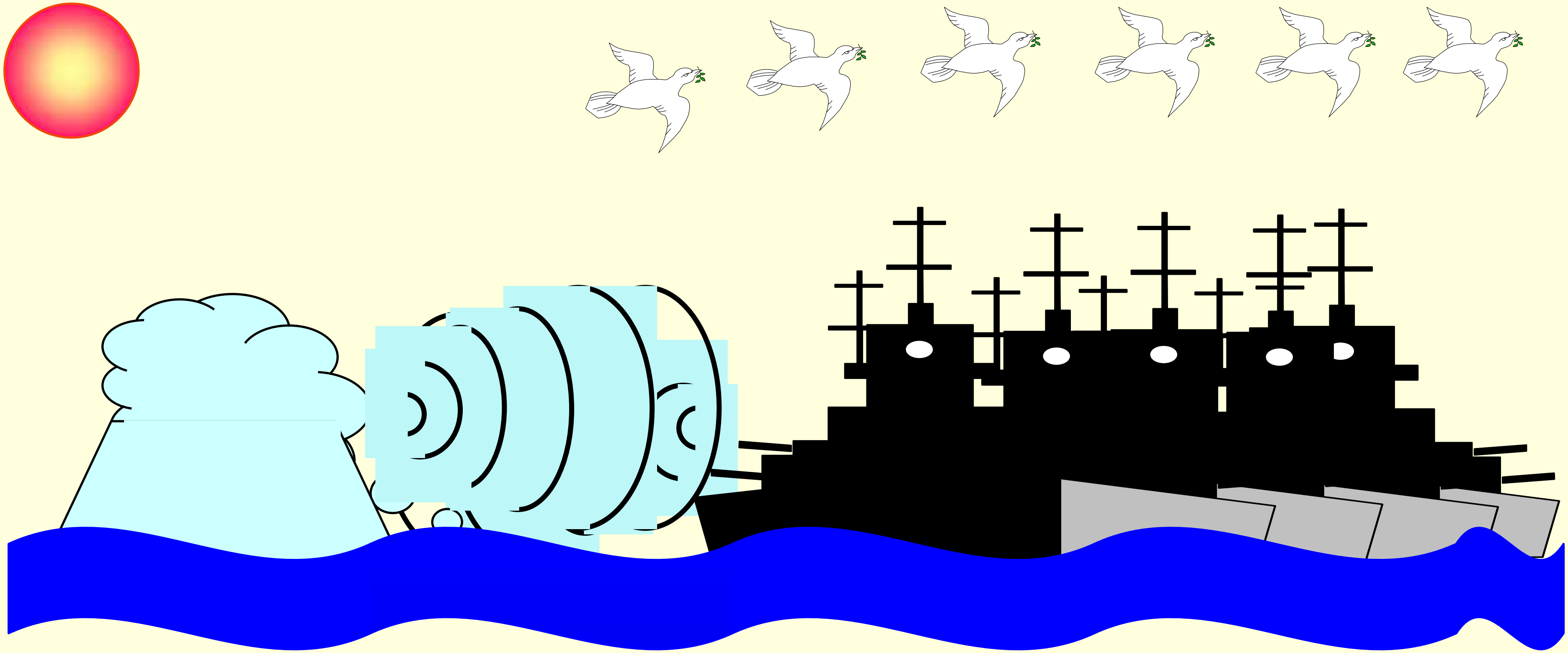


The measurement of the stress distribution of the driver's seat using piezoelectric sensor.



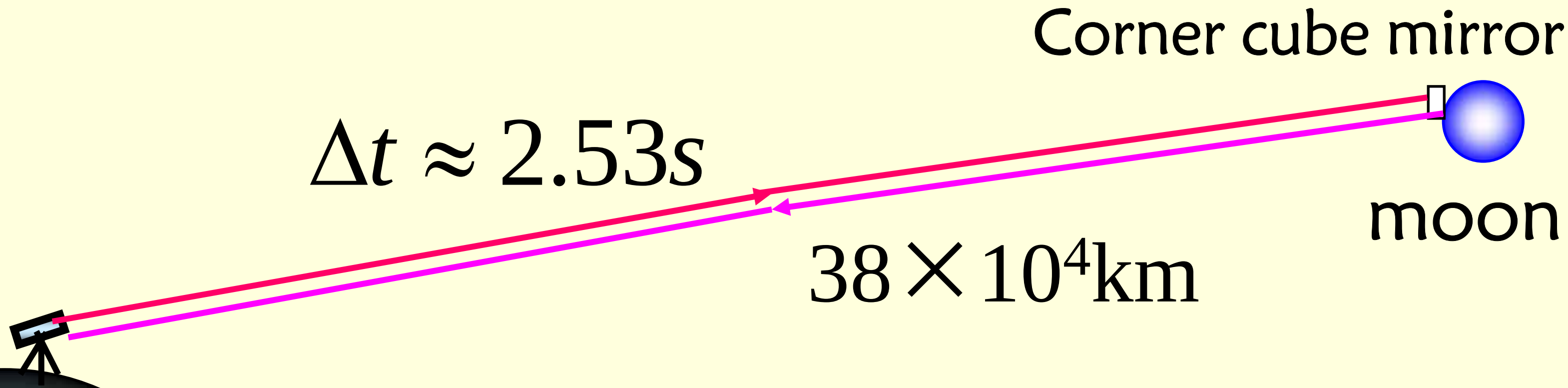
Drying apparatus

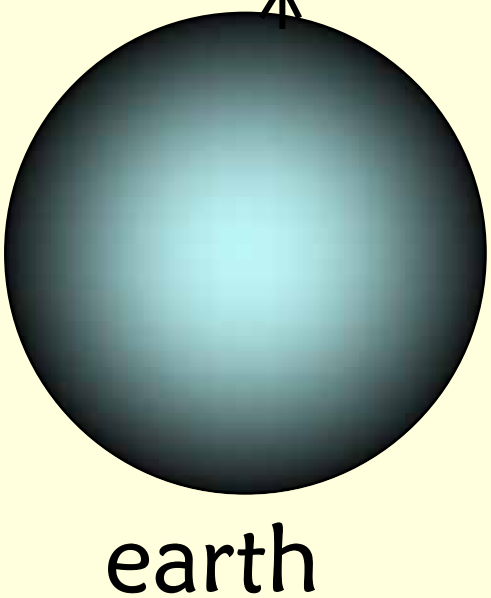




Electric means for non-electric quantity

The identification of the front barrier (berg, hidden rock, ship or fish stock) through sound wave.

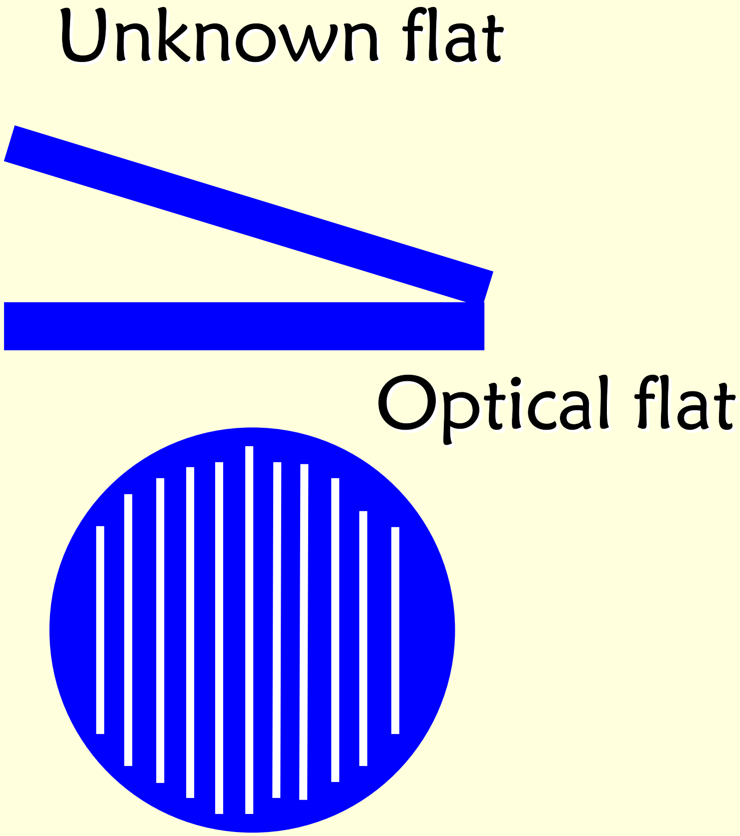


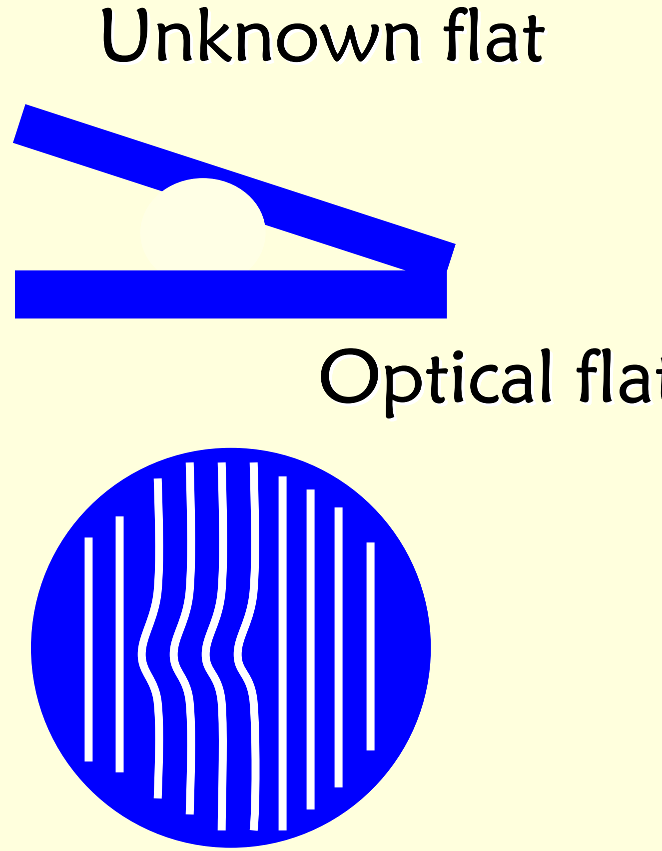


Optical means for non-optical quantity

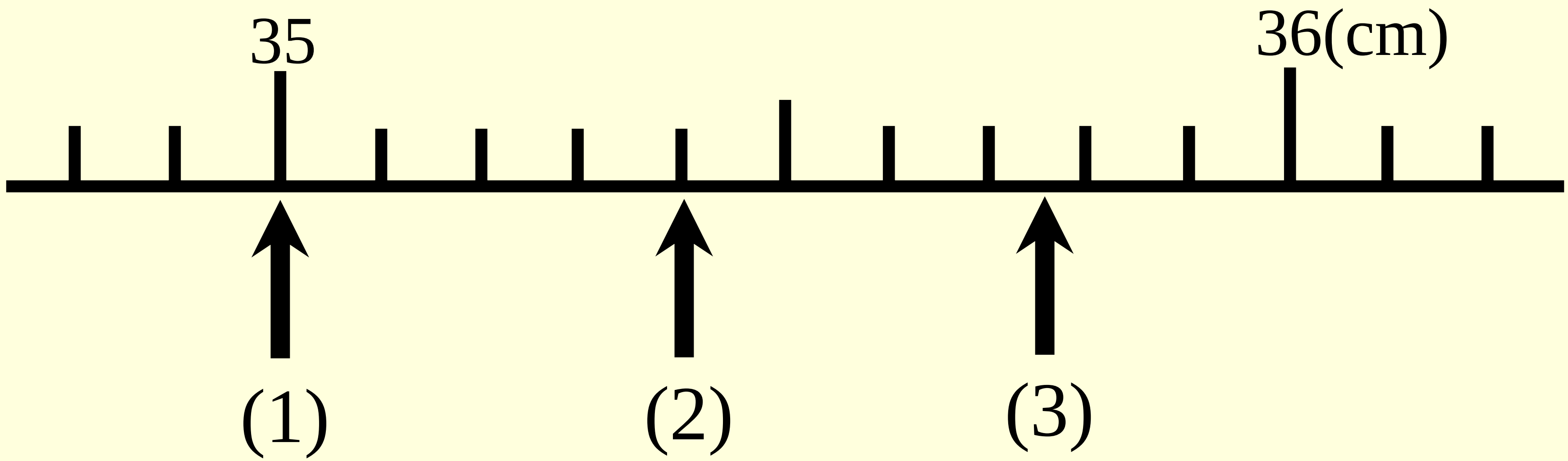
With known light velocity the distance between the earth and the moon can be obtained through measuring the time interval between signal sending and receiving. Laser is used to design ambulator because of its unidirection.

Interferometry: the base of modern fine measurement.





Significant Figures
有效数字



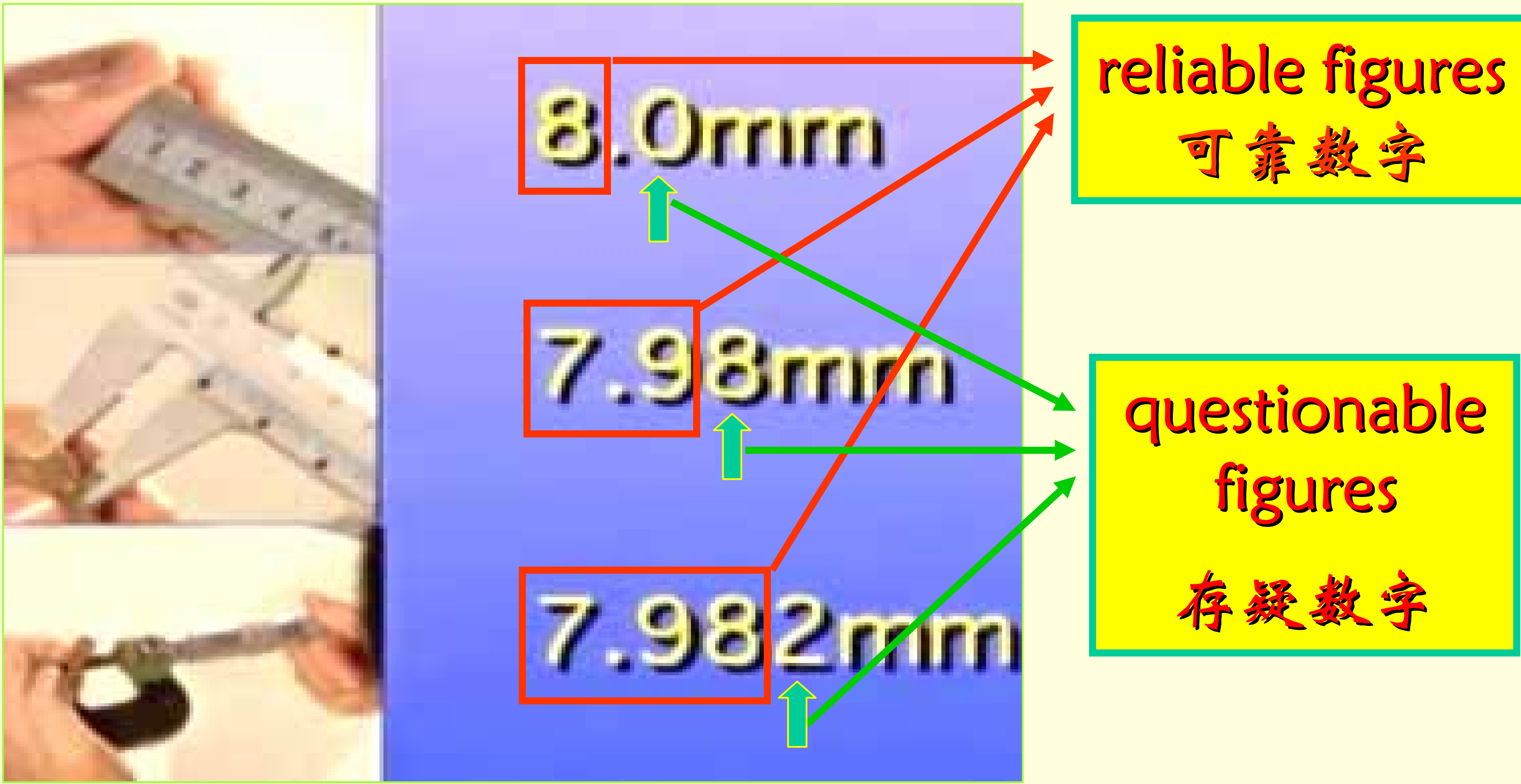
(1) 35.00cm not 35cm

(2) 35.40cm not 35.4cm

(3) A value between 35.7~35.8cm, as can be estimated as 35.76, 35.77 or 35.78.

reliable figures

questionable figures



All the reliable figures (certain figures) and questionable figure (uncertain figure) obtained in measurement are called significant figures.

A note on significant figures
对有效数字的说明

- Significant digits objectively reflect the measurement reality so it is not permitted to add or cut down significant digits at will. （有效数字位数不可随意增减）

- Only the last digit of a measured value is estimated. （只有最后一位为可疑位）

A note on significant figures

对有效数字的说明

- The number of significant figures has nothing to do with the position of decimal point. Zeros used to position the decimal point in number are not significant. （有效数字位数与小数点位置无关）

- The number of significant figures does not change in unit conversion.（在十进制单位换算过程中有效数字位数不变）

4.07cm → 3 significant figures

0.0407m → 3 significant figures

A note on significant figures

对有效数字的说明

- In the expression of the measurement results the last significant digit of the best value should be consistent with that of the uncertainty. （测量结果与不确定度的最后一个有效数位一致）

$$L = (100.00 \pm 0.06)\ \mathrm{cm}$$

- It is better to use scientific notation to express the numbers either too large or too small. （建议使用科学计数法）

Scientific Notation

科学计数法

Numbers expressed as some power of 10 multiplied by another number between 1 and 10 are said to be in scientific notation. To avoid potential confusion, it is always best to write numbers in scientific notation. Generally all the digits in a number written in scientific notation are considered significant (of course, don't write down the digits if they are not!).

Significant Figures in Calculation

有效数字运算规则

Addition and Subtraction

加减法运算

When adding or subtracting, the last significant digit of the result (sum or difference) is determined by the term whose last significant figure is on higher-order position.

$$\begin{array}{r} 10.\underline{1} \\ -\ \ 4.17\underline{8} \\ \hline 5.\underline{9}2\underline{2} \end{array}$$

This term has the least digits to the right of the decimal.

Round off the sum to 5.9, since the least number of decimal places found in the given terms is 1.

Multiplication and Division
乘除法运算

In multiplication or division, the number of significant figures in the result (product or quotient) equals that of the quantity which has the least number of significant figures regardless of the position of the decimal point.

$$\begin{array}{r} 4.178 \\ \times \quad 10.1 \\ \hline 4178 \\ 4178 \\ \hline 42.1978 \end{array}$$

This term has the least significant figures.

Round off the sum to 42.2, since the least number of significant figures found in the given terms is 3.

Involution and Extraction

乘方和开方

We use the same rule as used in multiplication or division, that is, the number of significant figures in result (power or root) equals the significant digits of the quantity which is taken power or root.

$$225^2 = 5.06 \times 10^4$$

$$\sqrt{225} = 15.0$$

Trigonometric Function

三角函数

In the calculation of trigonometric function, when we change the questionable (uncertain) digit in the independent variable the digit which changes most in the result is the last digit to be retained.

$$\sin 30^0 02^{'} = 0.500503748$$

$$\sin 30^0 03^{'} = 0.500755559$$

The first different digit is the forth decimal place, so

$$\sin 30^0 02^{'} = 0.5005$$

Logarithm 对数运算

When taking logarithms, retain in the mantissa (the number to the right of the decimal point in the logarithm) the same number of significant figures as there are in the number whose logarithm is taken.

$$\ln 19.83 = 2.9872$$

$$\lg 1.983 = 0.2973$$

$$\lg 0.1983 = \bar{1}.2973$$

Exponent
指数运算

When taking exponential calculation, the number of decimal digits in the result equals the number of decimal digits in the index number.

$$e^{9.14} = 9.32 \times 10^{3}$$

Constant 常数

Constants in the calculation don't affect the number of significant figures in the result; in other words, the constant should be considered containing infinitive significant figures.

- If a calculation involves a combination of mathematical operations having different significant figures, it is customary practice to carry out the calculation using all figures, and then go back and figure out how many significant figures the final result should have.

Rules for Rounding Numbers

有效数字修约规则

In rounding off a number to the correct number of significant figures:

- If the discarded digit (or insignificant digits) is less than 5 then the last digit to be retained is not changed (or the original number is rounded down). （四舍）

- If the discarded digit (or insignificant digits) is greater than 5 then the last digit to be retained is increased by one (or the original number is rounded up). （五入）

- If the discarded digit (or insignificant digits) is exactly 5 and the preceding significant figure is even, you round down; while if it is odd, you round up.

（末位凑偶法）

将下列数据取为四位有效数字

4.32749	4. 327
4.32751	4. 328
44.42501	44.43
4.32750	4.328
4.32850	4.328
4.51050	4.510

Rules for determining significant figures in reading are given below.

有效数字的读取规则

- Generally the significant figures of reading include the digit indicating the least division value and the estimated digit. The estimated digit does not need to be integer multiple of one tenth of the division value while it can be integer multiple of one fifth or half of the division value.（仪器最小分度为"1"时，读到最小分度后再估读一位）

- Sometimes the estimated digit is just the digit indicating the division value, for example the TW-5 physical balance. （最小分度不为"1"时，只读到最小分度位）

Rules for determining significant figures in reading are given below.

有效数字的读取规则

- For measuring devices with vernier the last significant digit is the division value of the vernier.（游标类量具，只读到游标分度值）

- There is no need to interpolate or estimate when using digital instrument since the last digit of the reading is the questionable (uncertain) digit.（数字式仪表无需估读，仪表所示最后一位即为欠准位）

Rules for determining significant figures in reading are given below.
有效数字的读取规则

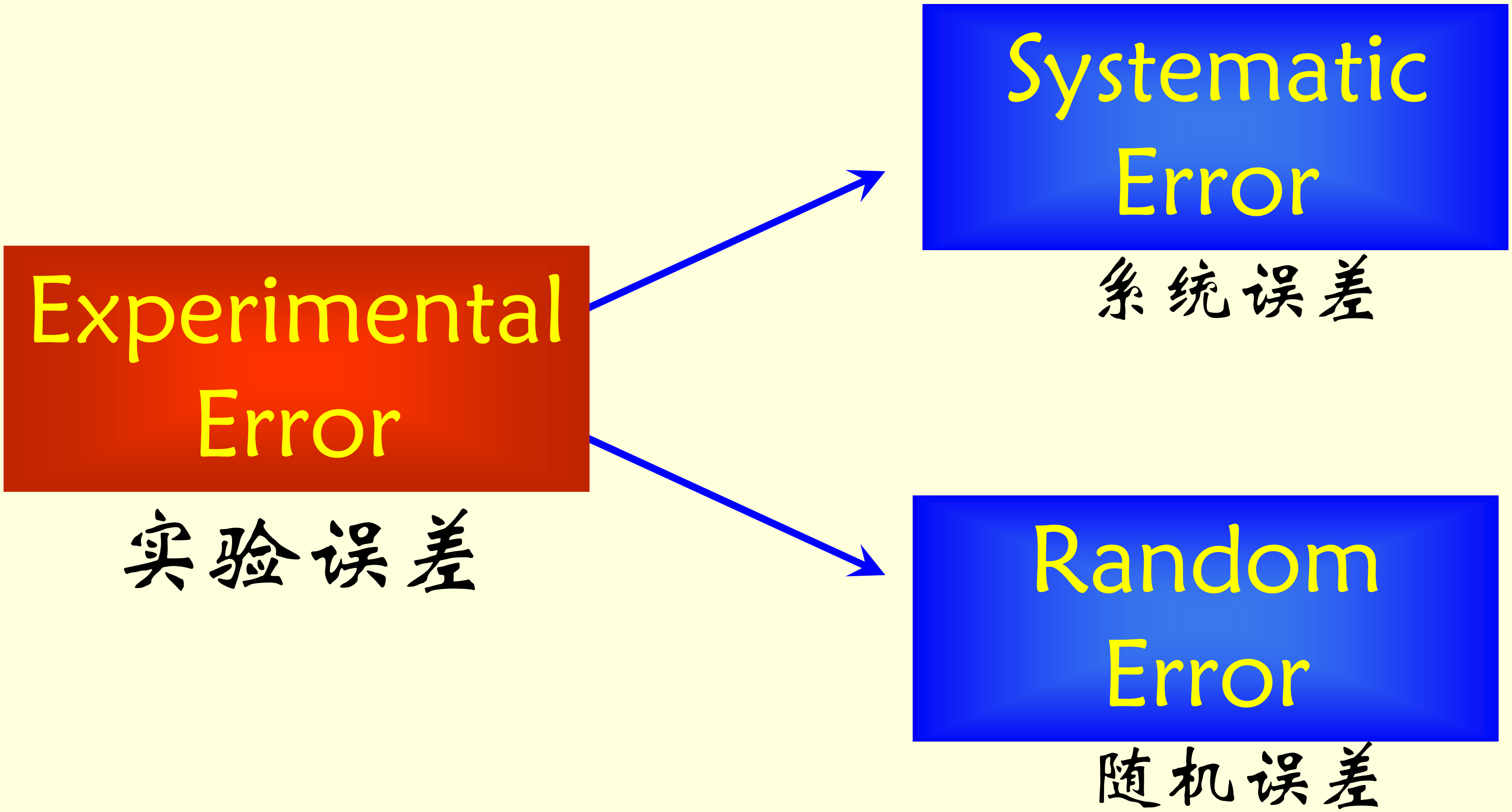
- When the indicating error of the instrument is given, the last significant digit should be correspondent with the error. (当给出仪器示值误差时，有效数字末位与示值误差一致)

- If the measured value is exactly an integral number we should position the digits with zero until the correct last significant digit. (如读数为整数需补"0"，一直补到可疑位)

Error

误差

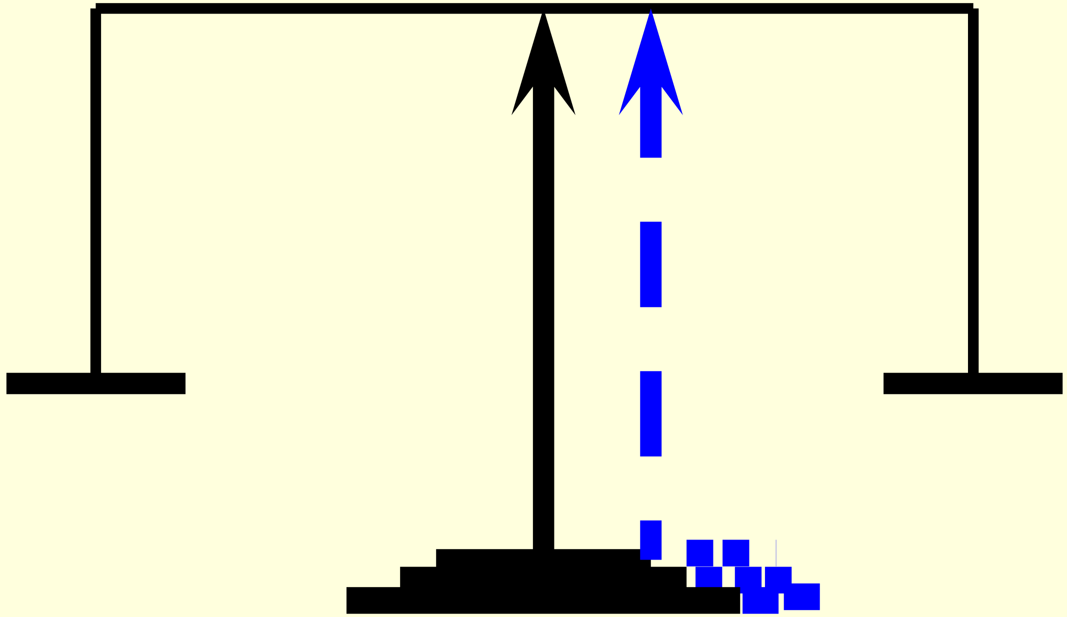
Error


Experimental Error
实验误差
Systematic Error
系统误差
Random Error
随机误差

Systematic errors
系统误差

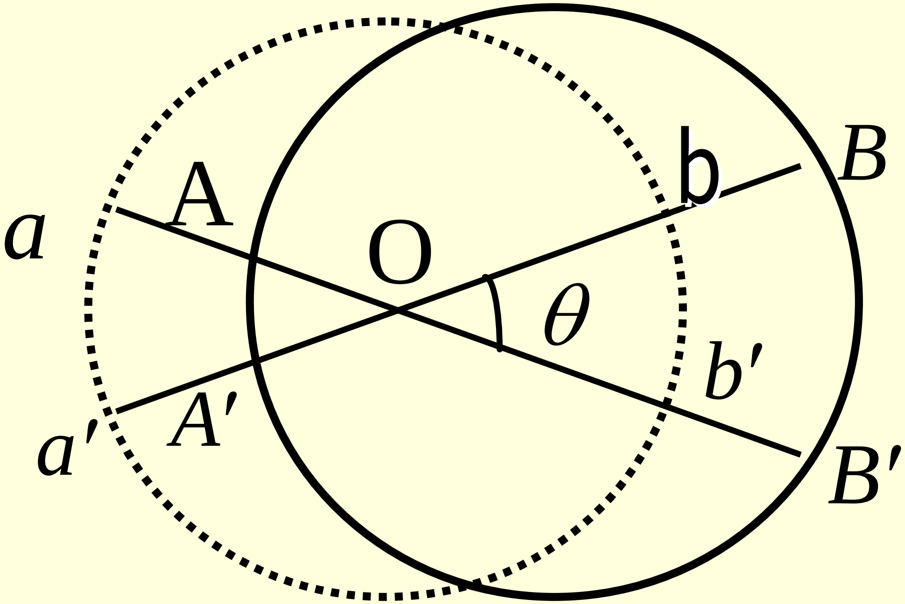
Systematic errors are those caused by the way in which the experiment was conducted. In other words, they are caused by the design of the system. Systematic errors cannot be eliminated by averaging. In principle, they can always be eliminated by changing the way in which the experiment was done. In actual fact though, you may not even know their existence.

Instrumental error



System error of the balance caused by unequal-arm.

Instrumental error



The arc length can reflect the angle if it is not eccentric, that is, $\widehat{aa'} = \widehat{bb'}$.

If it is eccentric the arc length cannot correctly reflect the angle, that is, $\widehat{AA'} < \widehat{BB'}$.

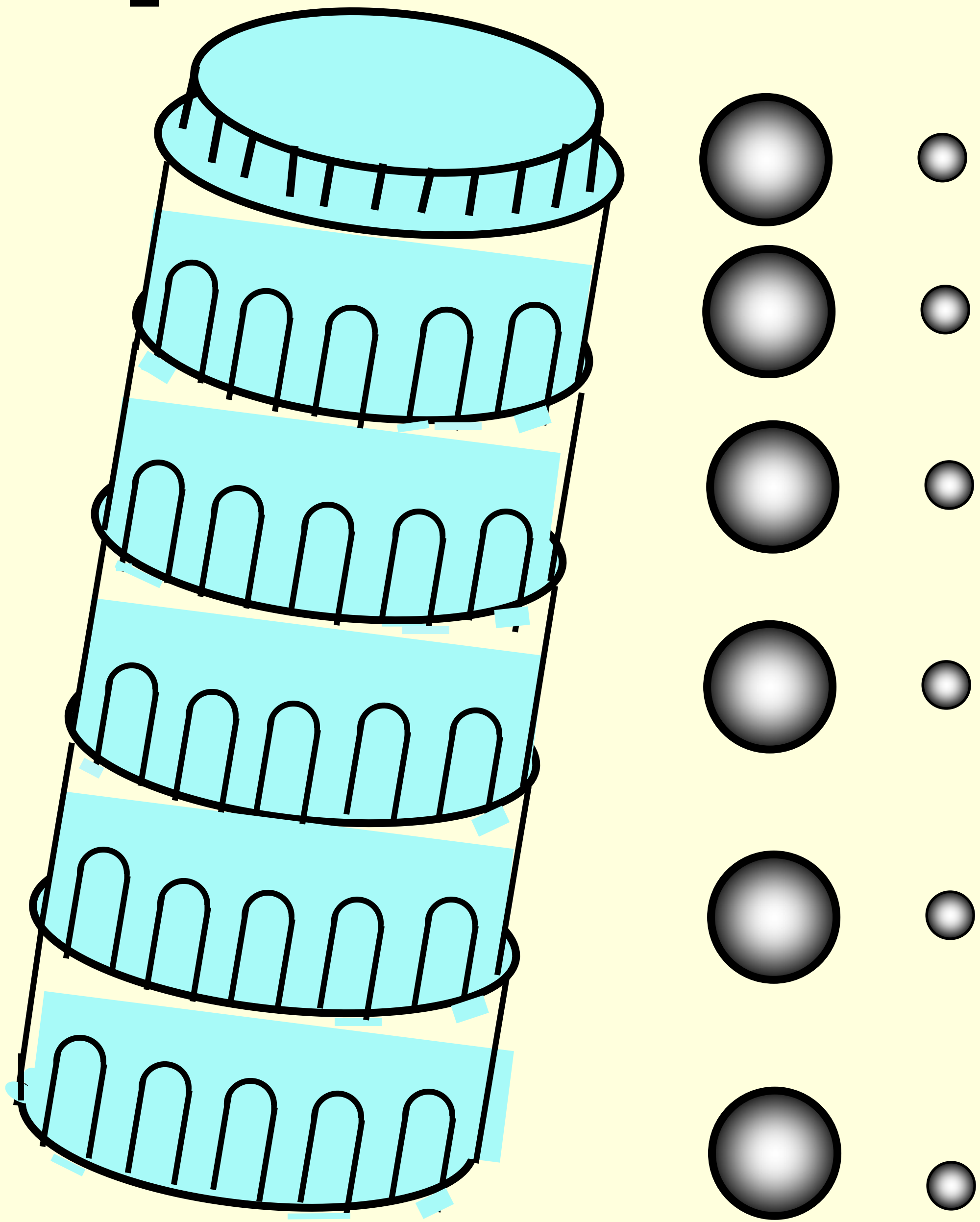
Theoretical error

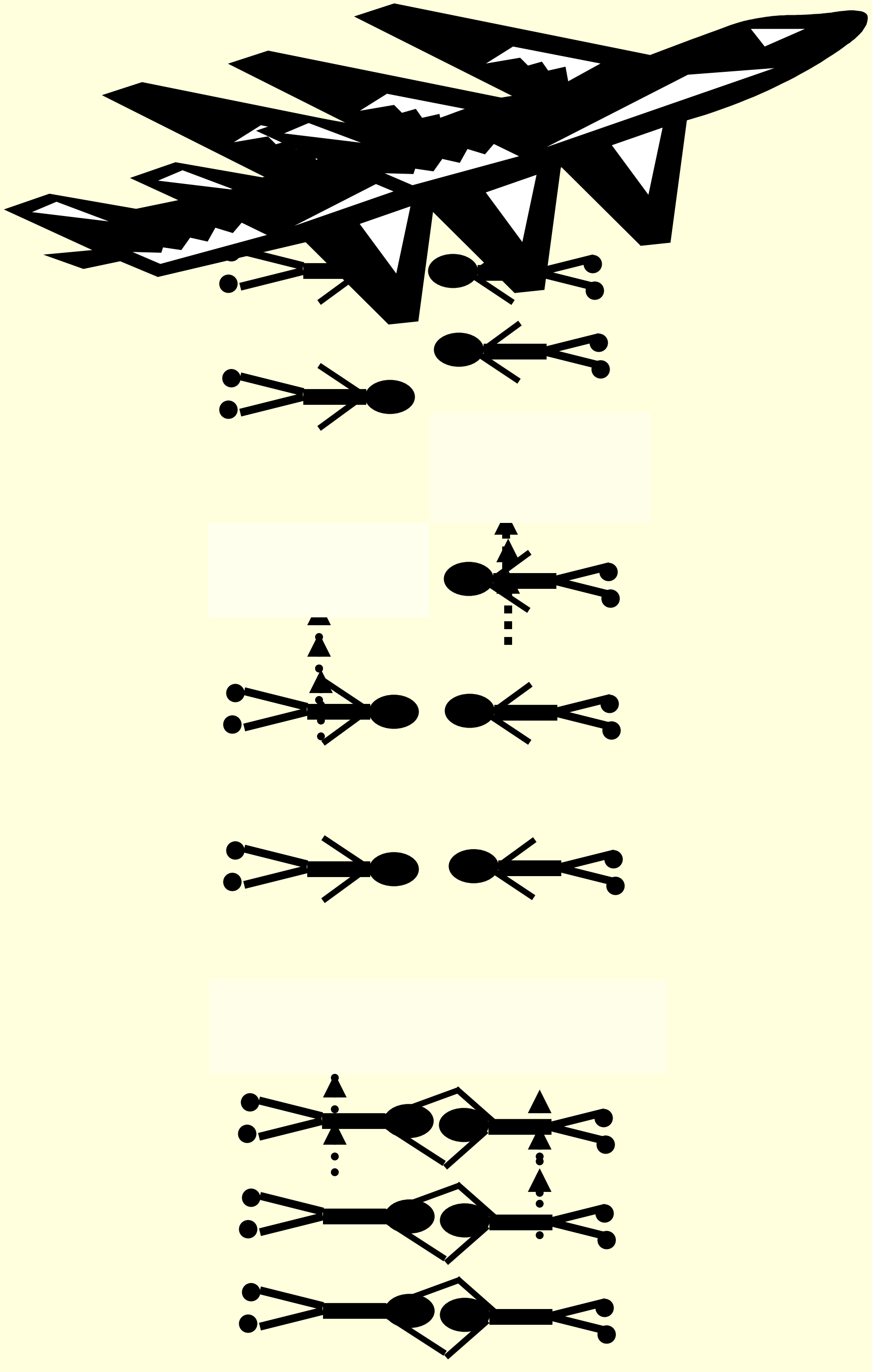
The system error caused in the course of theoretical derivation.

For example: $$B = \mu_0 nI$$

The magnetic leakage of the pipe wall can be ignored if the solenoid length is infinite.

Formula $h = \frac{1}{2} g t^2$ ignores the effect of gravitation etc.





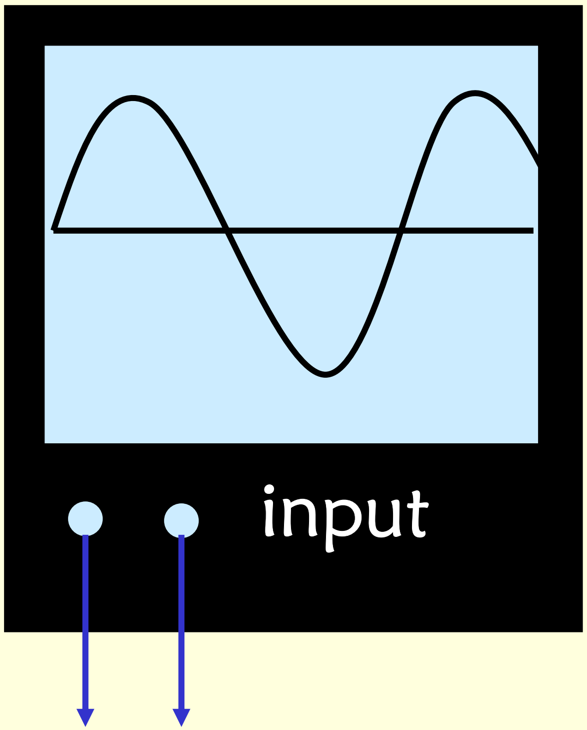
Artificial error

Psychological effects: The higher or lower interpolation of the measured value.

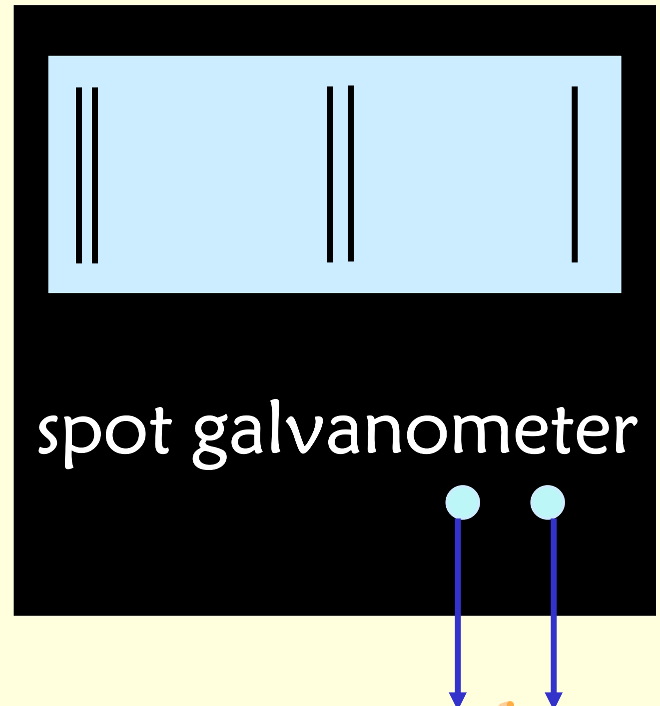
Physiology effects

- Audition
  - The discrimination of gamut (20Hz～20KHz).
  - The tone discrimination.
- Smell sense
- Color sense
- Visual sense

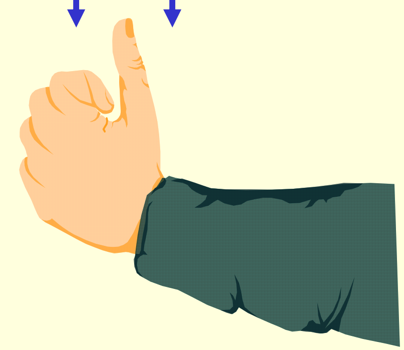
Environmental effects



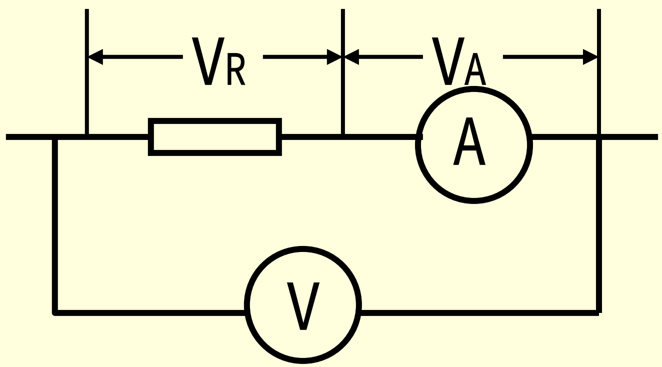
Interference of
commercial power



Static
interference

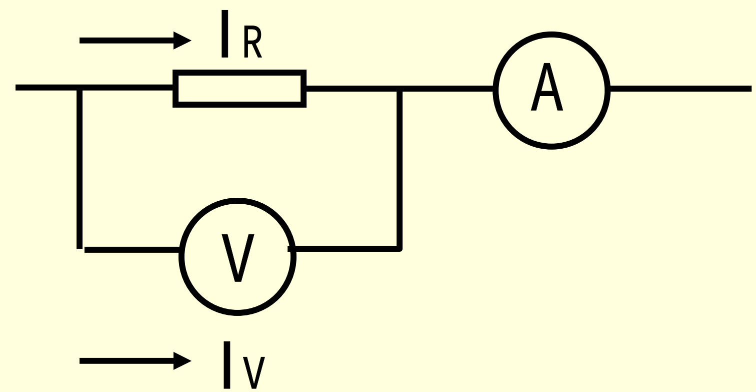


Methods



$$R = \frac{V}{I} = \frac{V_R + V_A}{I}$$

$$= \frac{V_R}{I} + \frac{V_A}{I} \rangle \frac{V_R}{I}$$



$$R = \frac{V}{I}$$

$$= \frac{V}{I_R + I_V} \angle \frac{V}{I_R}$$

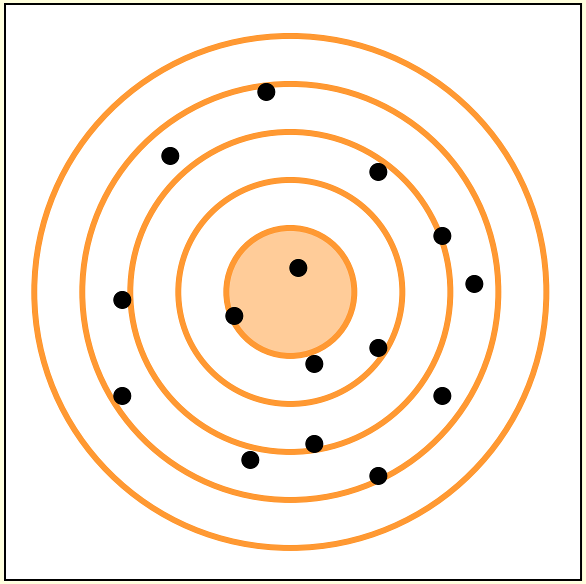
Random errors
随机误差

Random errors are unpredictable. They are chance variations in the measurement over which you as experimenter have little or no control. There is just as great a chance that the measurement is too big as that it is too small. Since the errors are equally likely to be high as low, averaging a sufficiently large number of results will, in principle, reduce their effect.

Accuracy、Precision and Exactness

Accuracy 正确度

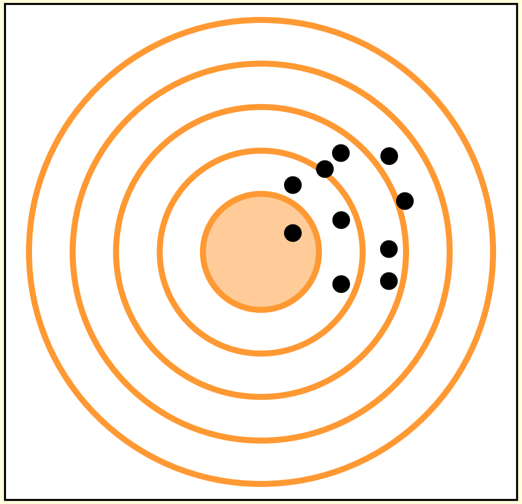
Accuracy indicates how close a measurement is to the true value or accepted value and it reflects the magnitude of system error of the result.



Accuracy、Precision and Exactness

Precision 精密度

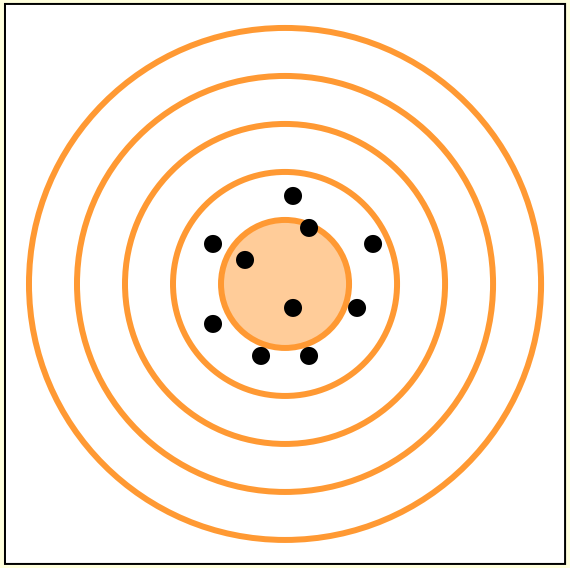
Precision indicates how close together or how repeatable the results are and it reflects the random error of the result. A precise measuring instrument will give very nearly the same result each time it is used.



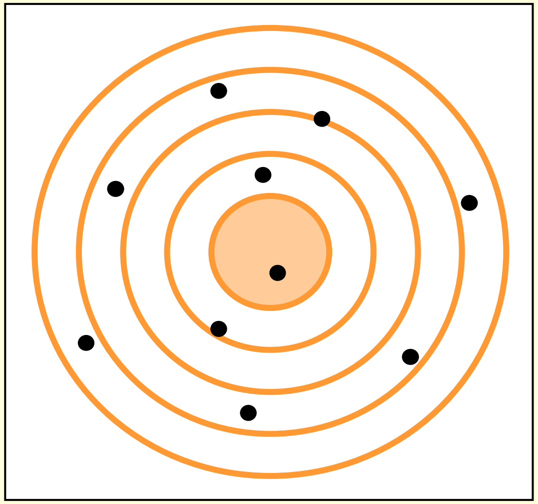
Accuracy、Precision and Exactness

Exactness 精确度

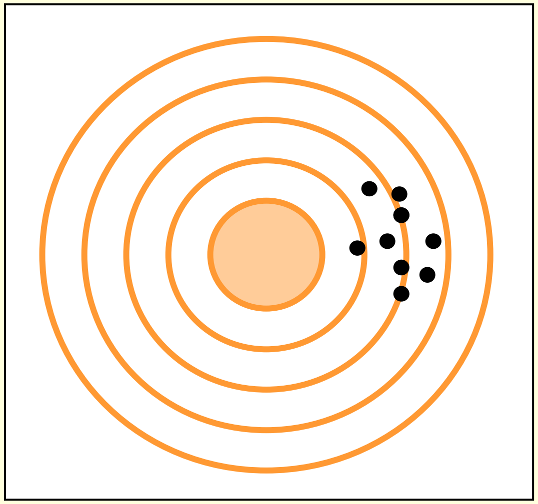
The measurement result being both accurate and precise is exact. The exactness comprehensively reflects systematic and random error.



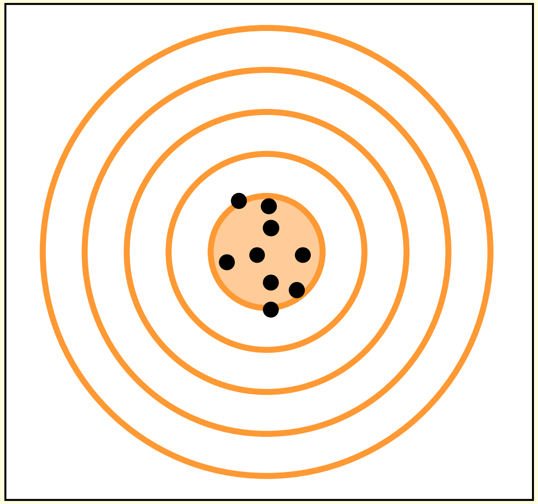
Accuracy、Precision and Exactness
正确度、精密度和精确度



Accurate but not precise



Precise but not accurate



Exact: both accurate and precise

Blunder errors
过失误差

Blunders errors may occur at any time, and are caused by carelessness on the part of the observer, booker or computer operator, e.g. pointing to the wrong light, misreading an instrument, incorrect booking, incorrect computer input, etc. Blunders will always occur sooner or later, but must never be allowed to occur undetected. For this reason, observing, booking and computing procedures must be designed to show them up.

Expressions of Experimental Error

- ✓ Absolute error 绝对误差
- ✓ Residual 残差
- ✓ Relative error 相对误差
- ✓ Percentage error 百分差

True Value and Measured Value

真值和测量值

Any physical quantity at any instant of time and in a position or in a specified state has an objective value which is called true value (or actual value) whose observation obtained in measurement is obviously called measured value.

Arithmetic mean

算术平均值

Arithmetic mean is the best estimated value of measured values.

$$\overline{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$$

Arithmetic mean is most reliable as can be approved by the law of probability.

Absolute error & Residual

绝对误差和残差

Absolute error means the difference between the measured or inferred value of a quantity x and its actual value x_0 .

$$\Delta x = x - x_0$$

Since we often take the arithmetic mean as the best value or actual value the actual value we define another expression as residual which represents the algebraic difference between each observation x and arithmetic mean $\overline{x}$.

$$\Delta x = (x - \overline{x})$$

Relative error （相对误差）

The relative error is defined as

$$E(x) = \frac{\Delta x}{\bar{x}}$$

Relative error is usually expressed as percentage.

$$E(x) = \frac{\Delta x}{\bar{x}} \times 100\%$$

Percentage error

百分误差

Percentage error is used to express the difference between measured value and the known theoretical value.

$$E_r = \frac{\text{measured value} - \text{theoretical value}}{\text{theoretical value}} \times 100\%$$

$$E_r = \frac{\text{测得值} - \text{理论值}}{\text{理论值}} \times 100\%$$

Treatment of Random Errors

- ✓ The expressions of single direct measurement

  单次测量结果的表示

- ✓ The expressions of multiple and direct metering under equal conditions

  多次等精度直接测量结果的表示

- ✓ Standard Deviation

  标准偏差

- ✓ Gaussian Distribution (Normal Distribution)

  高斯分布（正态分布）

The expressions of multiple and direct metering under equal conditions

多次等精度直接测量结果的表示

Standard Deviation 标准偏差

The standard deviation of the series with x_1, x_2,…, x_n can be defined as

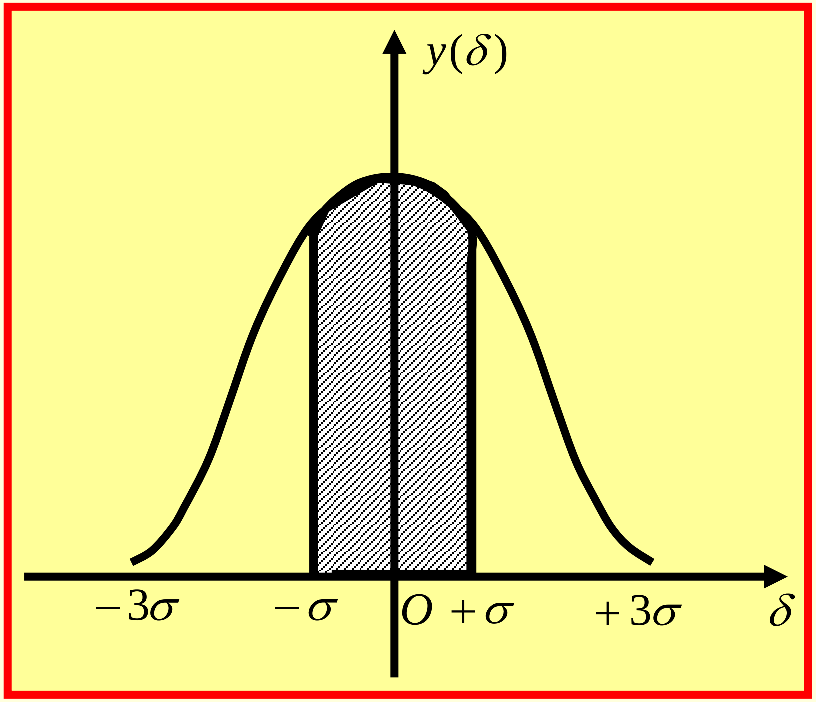
$$\sigma = \sqrt{\frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n-1}}$$

Bessel equation

σ is a parameter characterizing the degree of scatter of the measured value x_i with respect to the mean value $\bar{x}$.

Gaussian Distribution 高斯分布

The normal distributions are a very important class of statistical distributions. All normal distributions are symmetric and have bell-shaped density curves with a single peak.



Normal distribution curve

Features of normal distributions

- ✓Symmetric 对称性
- ✓Single-peaked 单峰性
- ✓Bounded 有界性
- ✓Compensatory 抵偿性

The Statistical Meaning of Standard Deviation

标准偏差的统计意义

68.3% of the random errors fall within the range $[-\sigma, +\sigma]$

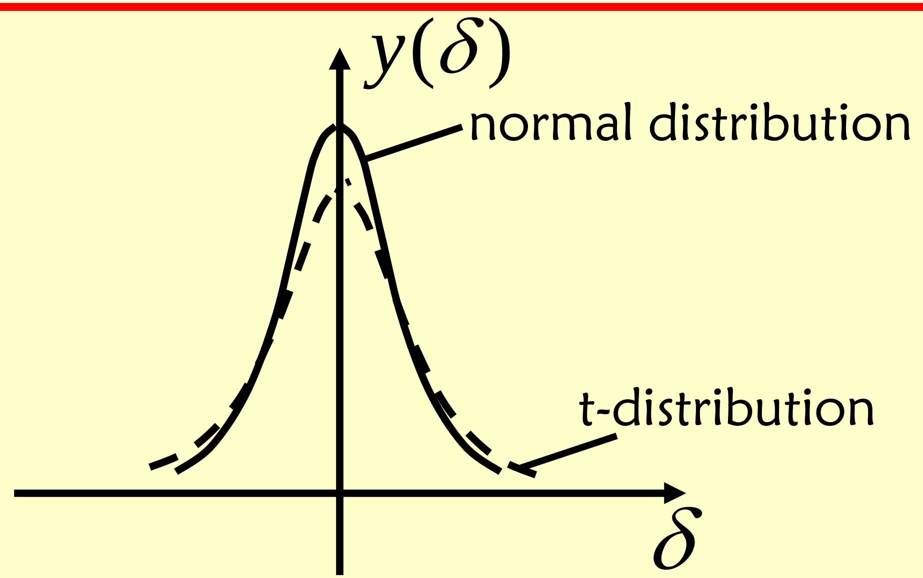
95.9% of the random errors fall within the range $[-2\sigma, +2\sigma]$

99.7% of the random errors fall within the range $[-3\sigma, +3\sigma]$

The percentage P=68.3% etc. is called fiducial probability and the range $[-n\sigma, +n\sigma]$ is called fiducial interval.

t-Distribution　　t－分布

When the times of measurements is small, for example less than 10 times, the error distribution of a series of observations will obviously deviate from normal distribution but follow the *t*-distribution or called student-distribution.



The curve of t-distribution has lower peak and is narrower in the upper part and wider in the lower part than normal distribution curve.

Uncertainty

不确定度

Uncertainty of measurement

测量不确定度

Uncertainty of measurement indicates the confidence level of the measured value and also indicates the dispersion rationally assigned to the measured value. It is a parameter, associated with the result of a measurement that defines the range of the values that could reasonably be attributed to the measured quantity. （合理赋予被测量之值的分散性）

The literal meaning is the extent to which the observations are questionable or uncertain because of the existence of measurement errors. （测量值的不确定程度）

The statistical meaning is the estimated range in which the measured quantity seems to occur. （对被测量真值所处范围的估计值）

Uncertainty
不确定度

Type A uncertainty

Evaluation:

Statistical methods

A类不确定度

以统计方法评定

Type B evaluation

Evaluation:

Non-statistical methods

B类不确定度

以非统计方法评定

Under t-distribution, that is when the fiducial probability is 95%, the Type A uncertainty is represented by standard deviation of the series of observations.

在t－分布下，A类不确定度近似等于测量列的标准偏差，置信概率为95%。

$$\Delta_A = \sigma$$

Type B Uncertainty of Direct Observations

直接测量的B类不确定度

Generally the indicating error is used to express the Type B uncertainty.

通常用仪器示值误差（限）或灵敏阈表示

$$\Delta_B = \Delta_m$$

The probability level of the indicating error is basically as much as 95%.

仪器示值误差的概率水平与95%相当。

Combining uncertainty of direct observations

直接测量的合成不确定度

Combining Uncertainty includes both Type A and Type B uncertainty and is treated statistically by means of root-sum-square method, given by complete equation of combining uncertainty

$$\sigma_x = \sqrt{\sum_i a_i^2 {\Delta_{A_i}}^2 + \sum_j b_j^2 {\Delta_{B_j}}^2 + R}$$

weighted factors

加权因子

covariance term

相关系数

Combining uncertainty of direct observations

直接测量的合成不确定度

In physical experiments, it is generally assumed that there are only one component of Type A uncertainty and one of Type B and that both the two types of components are independent of each other therefore the equation can be simplified as

$$\sigma_x = \sqrt{\Delta_A^2 + \Delta_B^2}$$

Usually the fiducial probability is taken as 0.95 for normal distribution while the actual times of measurement as n ($6 \le n \le 10$), then the combining uncertainty is

(p=0.95)

$$\sigma_x = \sqrt{\Delta_A^2 + \Delta_B^2}$$

$$\approx \sqrt{\sigma^2 + \Delta_m^2}$$

Notice

The fiducial probability is always taken as 0.95 unless for the especial statements. So generally the combining uncertainty can be calculated as

$$\sigma_x = \sqrt{\sigma^2 + \Delta_m^2}$$

★ When p=0.95, it is not necessary to mark the value of fiducial probability.

Expression of the measuring result

测量结果的表示

The result of directly measured quantity can be expressed as

$$X = \overline{x} \pm \sigma_x \text{ (unit)}$$

IMPORTANT

or as relative uncertainty given by

$$X = \overline{x}(1 \pm E(x)) \text{ (unit)}$$

where $E(x) = \sigma_x / \overline{x}$

Such expression gives the expected value of the quantity and the fiducial interval for a given fiducial probability.

Steps for evaluating the uncertainty of a direct observed quantity x

评定某直接测量量x不确定度的步骤

- Correct the known systematic errors of the data;

  修正数据中的可定系统误差

- Calculate the arithmetic mean $\overline{x}$ of the series of observations take it as the optimal value of the observations;

  计算测量列的算术平均值作为最佳值

Steps for evaluating the uncertainty of a direct observed quantity x

评定某直接测量量x不确定度的步骤

- Calculate the standard deviation of every observation σ ;

  计算测量列任意一次测量值的标准偏差

- Evaluate the indicating error Δ_m and take it as Type B uncertainty.

  求仪器的示值误差限，作为B类不确定度

Steps for evaluating the uncertainty of a direct observed quantity *x*

评定某直接测量量x不确定度的步骤

- Calculate the combining uncertainty

计算合成不确定度

$$\sigma_x = \sqrt{\sigma^2 + \Delta_m^2}$$

In single observation: $\sigma_x = \Delta_m$ （单次测量）

- Give the final result as:

写出最终表示式

$$\begin{cases} X = \bar{x} \pm \sigma_x \\ E(x) = \dfrac{\sigma_x}{\bar{x}} \times 100\% \end{cases}$$

Indication error
示值误差

The simplified margin of error or intrinsic error of meters or devices specified by national technical standards or calibration regulations, with suggested symbol Δ_m. It represents the absolute value of the maximal error between truth value and measured value on condition that the instruments or devices are correctly used.

Examples for indication error

- Vernier calipers---the division value of the caliper
- Spiral micrometer--- $\Delta_m = 0.004\,\text{mm}$ for the micrometers with range of 0~25mm.
- Balance---the minimal sensible mass
- Electricity meter--- $\Delta_m = \text{range} \times \text{K}\%$
- Resistance box--- $\Delta_m = R \times K\%$
- Devices with unknown indication error or degree of precision---half of the minimal division value
- Digital meter---the unit represented by minimal division indicated by the last digit of the reading

Sensitivity threshold

Sensitivity threshold refers to the minimal change of the quantity to be measured being enough to cause the sensible indicating change.

- The less the sensitivity threshold, the higher the sensitivity.

sensitivity threshold<indication error<least division value

- The sensitivity threshold will increase for the frequent use, when it is greater than indicating error limit the indicating error should be shown by sensitivity threshold.

Evaluations of uncertainty of indirect observations

间接测量不确定度的评定

Suppose that the relationship between indirectly measured quantity N and directly measured quantities $x_1, x_2, \ldots, x_n$ is given by

直接测量量

间接测量量

$$N = f(x_1, x_2, \cdots, x_n)$$

If the arithmetic means $\bar{x}_1, \bar{x}_2, \cdots, \bar{x}_n$ are taken as the optimal values of direct observations, it can be proved that the optimal value of the indirectly measured quantity should be

间接测量量的最佳值：

$$\bar{N} = f(\bar{x}_1, \bar{x}_2, \cdots, \bar{x}_n)$$

If the uncertainties of the directly measured quantities $x_1, x_2, \ldots, x_n$ are $\sigma_1, \sigma_2, \cdots, \sigma_n$ and the directly measured quantities are independent of each other then the uncertainty of the indirectly measured quantity N is given by

如果直接测量量$x_1, x_2, \ldots, x_n$的不确定度分别为$\sigma_1, \sigma_2, \cdots, \sigma_n$，并且各直接测量量之间彼此独立，则间接测量量$N$为

Uncertainty of the indirectly measured quantity *N*

$$\sigma_N = \sqrt{\sum_{i=1}^{n} \left(\frac{\partial f}{\partial x_i}\sigma_i\right)^2}$$

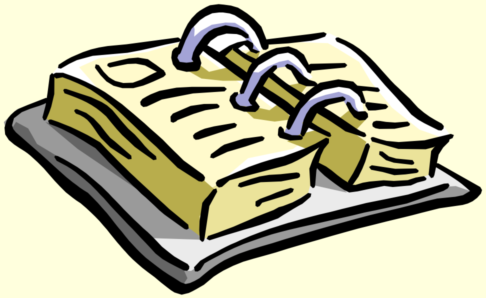
$$= \sqrt{\left(\frac{\partial f}{\partial x_1}\sigma_1\right)^2 + \left(\frac{\partial f}{\partial x_2}\sigma_2\right)^2 + \cdots + \left(\frac{\partial f}{\partial x_n}\sigma_n\right)^2}$$

Transfer Coefficient 传递系数

$$E(N) = \frac{\sigma_N}{\overline{N}} = \sqrt{\sum_{i=1}^{n} \left(\frac{\partial \ln f}{\partial x_i}\sigma_i\right)^2}$$

Uncertainty transferring formulae of major functions

Function	Transfer Formula	Function	Transfer Formula
$W = x \pm y$	$\sigma_W = \sqrt{\sigma_x^2 + \sigma_y^2}$	$W = kx$	$\sigma_W = k\sigma_x \quad \frac{\sigma_W}{W} = \frac{\sigma_x}{x}$
$W = x \cdot y$	$\frac{\sigma_W}{W} = \sqrt{(\frac{\sigma_x}{x})^2 + (\frac{\sigma_y}{y})^2}$	$W = k\sqrt{x}$	$\frac{\sigma_W}{W} = \frac{1}{2}\frac{\sigma_x}{x}$
$W = \frac{x}{y}$	$\frac{\sigma_W}{W} = \sqrt{(\frac{\sigma_x}{x})^2 + (\frac{\sigma_y}{y})^2}$	$W = \frac{x^k y^n}{z^m}$	$\frac{\sigma_W}{W} = \sqrt{k^2(\frac{\sigma_x}{x})^2 + n^2(\frac{\sigma_y}{y})^2 + m^2(\frac{\sigma_z}{z})^2}$
$W = \sin x$	$\sigma_W = \lvert\cos x\rvert \sigma_x$	$W = \ln x$	$\sigma_W = \frac{\sigma_x}{x}$



A rule of thumb

For the functions of addition and subtraction, the square of the combining uncertainty is the quadratic sum of every uncertainty component, while for the functions of multiplication and division, the square of the relative uncertainty is the quadratic sum of every relative uncertainty component.

Steps for evaluating the indirectly observed quantity

评定间接测量量不确定度的步骤

- Calculate the uncertainty of each directly measured quantity according to the steps in evaluation for the direct observations: （计算各直接测量量的不确定度）

$$\sigma_1, \sigma_2, \sigma_3, \ldots, \sigma_n$$

- Calculate the optimal value of the indirectly measured quantity: （计算间接测量量的最佳值）

$$\overline{N} = f(\overline{x}_1, \overline{x}_2, \cdots, \overline{x}_n)$$

Steps for evaluating the indirectly observed quantity

评定间接测量量不确定度的步骤

- Calculate the uncertainty and relative uncertainty of the indirectly measured quantity *N*, using the composite equations for uncertainty.

间接测量量的不确定度

$$\sigma_N = \sqrt{\sum_{i=1}^{n} \left(\frac{\partial f}{\partial x_i}\sigma_i\right)^2}$$

$$E(N) = \frac{\sigma_N}{\overline{N}} = \sqrt{\sum_{i=1}^{n} \left(\frac{\partial \ln f}{\partial x_i}\sigma_i\right)^2}$$

Steps for evaluating the indirectly observed quantity

评定间接测量量不确定度的步骤

- Give the final result as （给出最终结果）

$$\begin{cases} N = \overline{N} \pm \sigma_N \\ E(N) = \dfrac{\sigma_N}{\overline{N}} \times 100\% \end{cases}$$

important

The significant figures of the uncertainty and its optimal value obey the same rules of fetching bits as for the direct observations.

Rule for fetching bits 取位规则

Generally the significant figures of uncertainty or relative uncertainty have one or two digits.

- If the most significant figure is larger than or equals 3, the uncertainty should have one digit;

$$D \pm \sigma_D = (0.136 \pm 0.008)mm$$

- If the most significant figure is less than 3, the uncertainty should have one or two digits;

$$D \pm \sigma_D = (0.136 \pm 0.012)mm$$

Rule for fetching bits 取位规则

- We can retain one more digit for the uncertainty of some results with more precision and more significance, so can we for the measured value.

For example the Plank constant:

h = (6.626076 ± 0.000036) ×10-34 J·s

- The last significant figure of the measuring result should be on the same digit with that of the uncertainty.

Criterion for micro-uncertainty

- When several components contribute to the combining uncertainty, maybe only one or two of them are mainly effective. So the uncertainty terms contributing least to the combining uncertainty can be ignored.

- Generally the uncertainty term which is less than one third of the largest term, that is, the least quadratic term which is less one ninth of the largest quadratic term can be ignored.

Methods for data processing

数据处理基本方法

Methods for data processing

- Schedule method

  列表法
- Graphing method

  作图法
- Successive difference

  逐差法
- Least-square procedure

  最小二乘法

Schedule method 列表法

Table1. Relationship between resistance and temperature

$t\,/\,°C$	24.0	30.0	40.0	50.0	60.0	70.0	80.0	90.0
$R\,/\,\Omega$	3.500	~~3.581~~ 3.582	3.724	3.875	4.025	4.170	4.302	4.443

表头 → 物理量&单位 → 记数 → 修改 → 参数&说明

Steps for graphing method 作图法的步骤

1、Choose proper coordinate paper
选取适当的坐标纸

2、Definite coordinate axis 确定坐标轴

3、Definite gradation 完成标度

4、Mark experiment points 表明实验点

5、Plot curve 连线

6、Give explanatory text 标明图注

Coordinate paper

坐标纸

Coordinate axis

坐标系

Gradation

标度

Experiment points

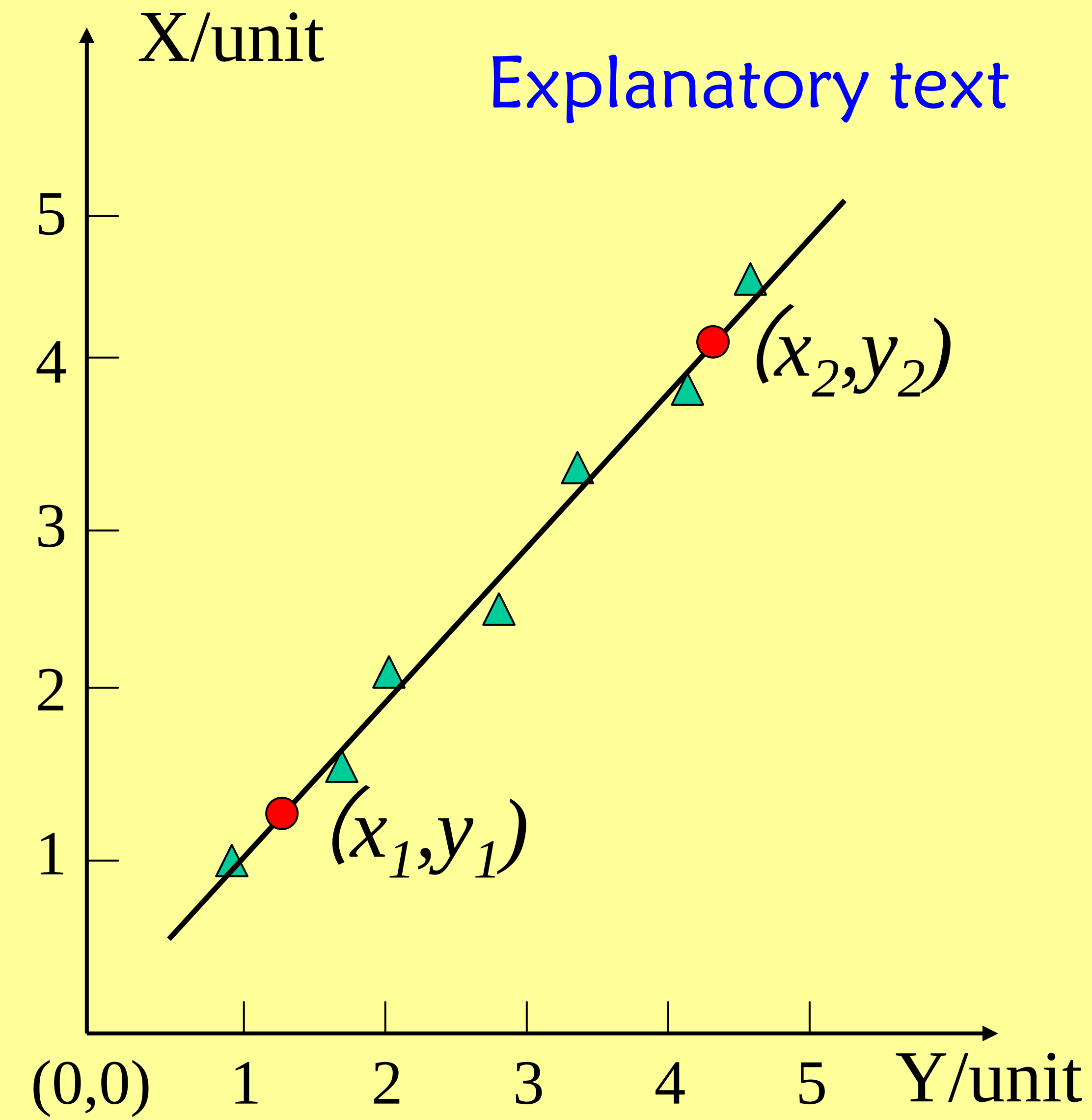
实验点

Curve

曲线

Explanatory text

图注



Slope 斜率

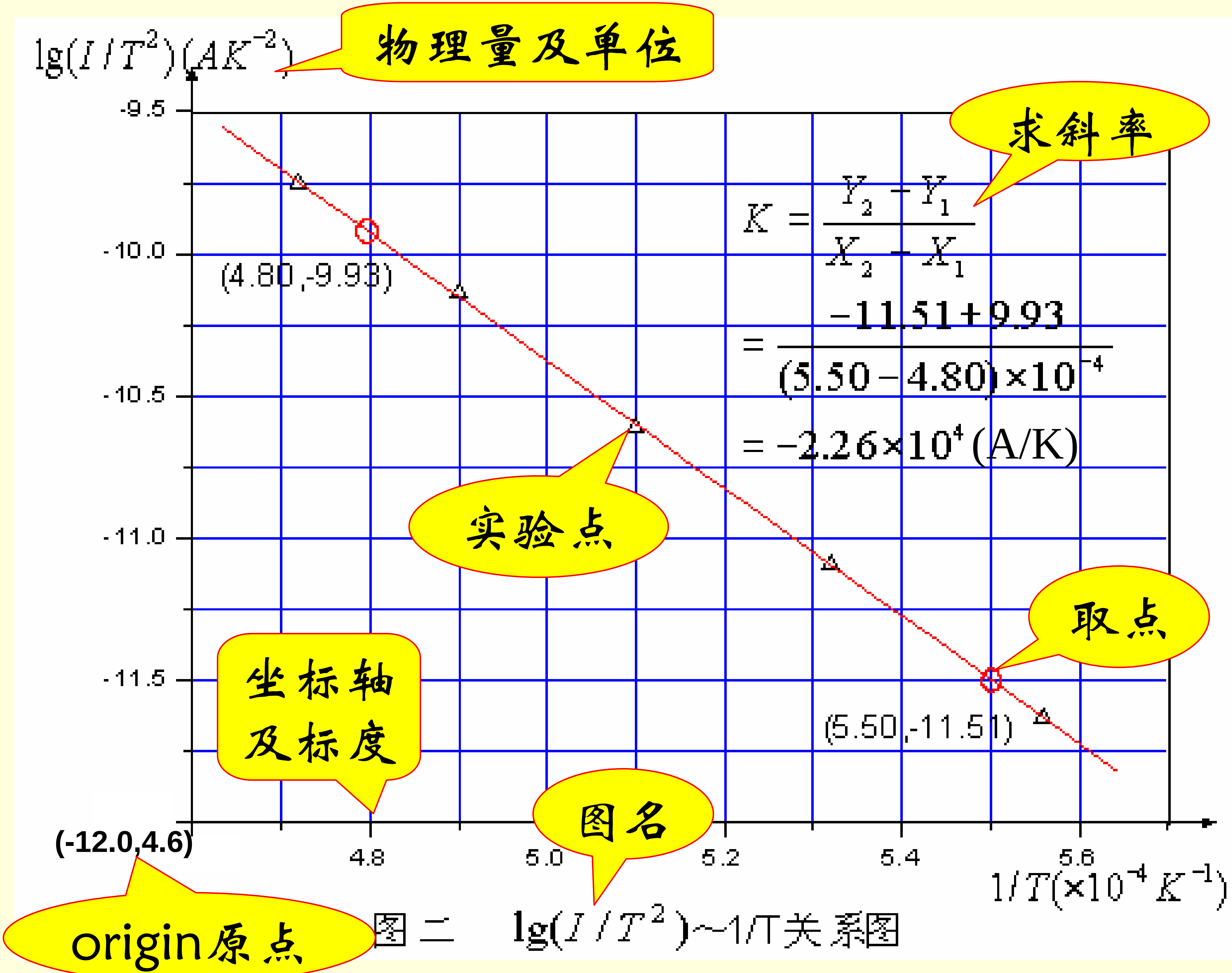
For the line $y=ax+b$, the slope is given with known points:

$$b=\frac{y_2-y_1}{x_2-x_1}(unit)$$

Intercept 截距

The intercept is written as:

$$a=\frac{x_2y_1-x_1y_2}{x_2-x_1}(unit)$$



图二 $\lg(I/T^2)$~1/T关系图

作图法的优点和局限

- 物理量之间的对应关系和变化趋势形象直观
- 可直观反映复杂函数关系
- 直观反映实验数据中的极值、拐点及周期性变化
- 有取平均的效果，通过内差和外推可得到其它数据

- 受图纸大小的限制
- 连线有很大的主观随意性
- 受图纸本身的均匀性、准确程度和线段粗细的影响

Successive difference 逐差法

Condition: The dependent variable varies linearly with the independent variable and changes of the later one are uniformly-spaced.

应用条件：因变量与等间距变化的自变量之间为线性关系。

$$\begin{cases} x: x_1, x_2, \ldots\ldots, x_n, x_{n+1}, \ldots\ldots, x_{2n} \\ y: y_1, y_2, \ldots\ldots, y_n, y_{n+1}, \ldots\ldots, y_{2n} \end{cases}$$

$$\begin{cases} \Delta y_1 = y_{n+1} - y_1 \\ \vdots \\ \Delta y_n = y_{2n} - y_n \end{cases} \qquad \overline{\Delta y} = \frac{1}{n}\sum_{i=1}^{n} \Delta y_i$$

3. 逐差法

Least-square procedure 最小二乘法